on the slip plane, the majority being in the dislocation crack. We may thus write

$$n = n'.$$

By substituting the right-hand side of equation (8) for n in equation (7), we get

$$\frac{\pi}{4G}(1 - \nu)(\sigma - \sigma_0)d\left\{\sigma + \sigma \sin\left(\theta - \frac{\pi}{4}\right) - p_c\right\} = 4\gamma'. \tag{9}$$

Now consider the value of σ which will be the flow stress at which the brittle fracture occurs. Armstrong and others [11] have shown that the flow stresses, σ_f, at equal strains of many metals and alloys obey the Petch relationship

$$\sigma_f = \sigma_0 + kd^{-\frac{1}{2}}, \tag{10}$$

where k is a constant for a particular metal and strain. The amount of strain occurring prior to brittle fracture is generally small, and any variations due to grain size are likely to be of secondary importance. We may thus put

$$\sigma = \sigma_0 + kd^{-\frac{1}{2}}, \tag{11}$$

since σ will be the flow stress at fracture.

Since $(\theta - \pi/4)$ is positive the value of $\sigma \sin(\theta - \pi/4)$ will have a maximum value of σ and a minimum value of 0, so we may write

$$\sigma + \sigma \sin\left(\theta - \frac{\pi}{4}\right) = \alpha\sigma, \tag{12}$$

where $1 \leqslant \alpha \leqslant 2$. Substituting appropriate terms from equations (11) and (12) in equation (9) gives

$$\frac{\pi}{4G}(1 - \nu)kd^{\frac{1}{2}}(\alpha\sigma - p_c) = 4\gamma'.$$

Rearranging, we get

$$p_c = \alpha\sigma - \left\{\frac{16G\gamma'}{\pi(1 - \nu)k}\right\} d^{-\frac{1}{2}} \tag{13a}$$

or

$$p_c = \alpha\sigma_0 - \left\{\frac{16G\gamma'}{\pi(1 - \nu)k} - \alpha k\right\} d^{-\frac{1}{2}}. \tag{13b}$$

Now the terms in the brackets are substantially constant, for constant temperature, so we may write

$$p_c = \alpha\sigma - \beta d^{-\frac{1}{2}} \tag{14a}$$

or

$$p_c = \alpha\sigma_0 - \beta' d^{-\frac{1}{2}}. \tag{14b}$$

If the hydrostatic pressure, p, exceeds the critical value, p_c, then ductile fracture will occur. Conversely, brittle fracture will occur if p is less than p_c.

The constants cannot be accurately determined theoretically in view of the assumption that a two-dimensional analysis is applicable to a three-dimensional problem. However, it is expected that values of α, β and β' derived from known values of the various parameters will be of the right order of magnitude.

3. THE BRITTLE-DUCTILE TRANSITION PRESSURE IN ZINC

The work of Pugh[1,3] shows that brittle fracture in zinc just below the transition pressure occurs after approximately 5 per cent strain and that the transition pressure for the particular grain size used ($d \simeq \frac{1}{2}$ mm) was 7.5 kg/mm^2. The results of Armstrong and others[11] show that for 5 per cent strain k is 0.76 kg/mm$^{\frac{3}{2}}$ and σ_0 is 5.8 kg/mm^2. We may thus write from equation (14b)

$$7.5 = 5.8\alpha - 1.4\beta'. \qquad (15)$$

From Armstrong and others[11] the transition at atmospheric pressure would appear to be at a $d^{-\frac{1}{2}}$ value of approximately 2.8 mm$^{-\frac{1}{2}}$. From this information we get

$$0 = 5.8\alpha - 2.8\beta'. \qquad (16)$$

At this point it should be noted that zinc slips only on (0001), so that $\theta = 0$ and in our two-dimensional model α should be approximately 1.7. However, the value of α will be modified by the three-dimensional nature of the problem, and Sack's analysis[9] suggests a value which will be greater by a factor of approximately 1.6, i.e. 2.7. Zinc also twins very readily, and it is known that cleavage fractures can originate at twin intersections. This will also modify the value of α. The work of Armstrong and others[11] suggests an increase in k and a decrease in σ_0 when twin boundaries are counted as well as grain boundaries. A simple calculation shows that the slip systems in zinc are unlikely to be affected by the applied hydrostatic pressure.

From equations (15) and (16) we get

$$\alpha = 2.59$$

$$\beta' = 5.36 \text{ kg/mm}^{-\frac{3}{2}}.$$

The value of γ' corresponding to β' is 1780 ergs/cm^2. This is about double the true surface energy of zinc and is lower than expected. The value of α is also lower ~~higher~~ than expected. However, the approximate data used would probably account for this.

4. EFFECT OF TEMPERATURE ON THE BRITTLE-DUCTILE TRANSITION PRESSURE

In equation (13) the main temperature-dependent parameters are k, σ_0 and γ'. The parameter k, which is a measure of the dislocation locking strength and is thus subject to thermal activation, can be roughly approximated to $k_0 e^{-\epsilon T}$, where k_0 and ϵ are constants. The variation of σ_0 with temperature, at least in the case of b.c.c. metals, stems from a large Peierls-Nabarro stress. In the case of h.c.p. metals, the Peierls-Nabarro stress is small and so the temperature variation of σ_0 can be ignored. Heslop and Petch[12] have shown that the Peierls-Nabarro stress can be represented by $\sigma_{00} e^{-\chi T}$, where σ_{00} and χ are constants. The whole of σ_0 will obey this type of relation approximately. The temperature dependence of γ' is unknown and in this case will be assumed to be temperature independent.